Indian Statistical Institute, Bangalore
B. Math (II)
First semester 2009-2010
Mid-Semester Examination : Statistics (I)

Date: 18-09-2009 **Maximum Score 70** **Duration: 3 Hours**

1. To establish a standard for parachute design, a researcher recorded the following fill times, in seconds, for 27 standard parachutes, obtained under controlled test conditions.

.59	.38	.47	.43	.44	.37	.43	.37	.27	.54	.39	.89	.48	.52
.51	.49	.38	.38	.23	.44	.40	.36	.33	.82	.51	.44	.37	

   (a) Make a stem and leaf plot of these data.

   (b) Find the sample mean $\overline{x}$.

   (c) Find the sample standard deviation s.

   (d) Find the sample median m.

   (e) Find $100p$ percentiles for $p = 0.25$ and 0.75.

   (f) Find the first and third quartiles.

   (g) What proportion of the data lies within $\overline{x} \pm 3s$?

   (h) Draw the box plot and identify the outliers.

   (i) Decide on trimming fraction just enough to eliminate the outliers and obtain the trimmed mean $\overline{x}_T$.

   (j) Also obtain the trimmed standard deviation s_T.

   $[4 + 2 + 2 + 2 + 4 + 2 + 2 + 5 + 2 + 3 = 28]$

   (k) Between the box plot and the stem and leaf plot what do they tell us about the data set?

   [04]

2. The following data are from a study of degradation of insulin by the liver cells of rats under two different experimental conditions.

EC 1	30.2	34.2	33.7	7.8	23.9	9.9	31.3	27.3	17.0	26.7	21.4
EC 2	19.5	40.4	24.1	12.9	18.5	25.0	29.5	6.6			

   EC = Experimental Condition

   Make a schematic diagram of box plots of two sets observations in the data, interpret. Describe any differences in the distributions.

   [12]

[PTO]

3. In a large University a section of students is opposed to 'dress code' arguing that the students anyway follow certain self imposed norms of decency and 'dress code' curbs individual freedom. A random sample of 200 students is drawn. Build a model to obtain the distribution of number of students opposed to 'dress code' in the random sample. What would be a good guess for the proportion of students opposed to 'dress code' in the entire University? If there were 160 students in the sample opposed to 'dress code' what would be the numerical value of your guess?

$[3 + 5 + 2 = 10]$

4. Consider a telephone operator who, on the average, handles five calls every three minutes. What is the probability that there at least two calls in the next minute? What is the probability that there would be no calls in next two minutes. If there is a call at time t_1 and T is the waiting time for the next call find $P[T > t]$. What is the distribution of T?

$[3 + 3 + 5 + 1 = 12]$

5. The independent random variables $X_1,\ X_2, ...,\ X_n$ have common distribution specified by *probability density function*

$$f(x|\theta) = \begin{vmatrix} \theta x^{\theta-1} & if\ 0 \leq x \leq 1 \\ 0 & otherwise \end{vmatrix}$$

where $\theta > 0$. Obtain *method of moments estimator* for θ.

[08]

6. The independent random variables $X_1,\ X_2, ...,\ X_n$ have common distribution specified by

$$P(X \leq x|\alpha, \beta) = \begin{vmatrix} 0 & if\ x < 0 \\ \left(\frac{x}{\beta}\right)^{\alpha} & if\ 0 \leq x \leq \beta \\ 1 & if\ x > \beta \end{vmatrix}$$

where α, β are positive. Obtain *maximum likelihood estimators* for α, β. The length of cuckoos' eggs found in hedge sparrow nests can be modelled with this distribution.

[10]